

Complete hyperfine Paschen-Back regime at relatively small magnetic fields realized in Potassium nano-cell

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Abstract – A one-dimensional nano-metric-thin cell (NC) filled with potassium metal has been built and used to study optical atomic transitions in external magnetic fields. These studies benefit from the remarkable features of the NC allowing one to use $\lambda/2$ - and λ -methods for effective investigations of individual transitions of the K D_1 line. The methods are based on strong narrowing of the absorption spectrum of the atomic column of thickness L equal to $\lambda/2$ and to λ (with $\lambda = 770$ nm being the resonant laser radiation wavelength). In particular, for a π -polarized radiation excitation the λ -method allows us to resolve eight atomic transitions (in two groups of four atomic transitions) and to reveal two remarkable transitions that we call Guiding Transitions (GT). The probabilities of all other transitions inside the group (as well as the frequency slope versus magnetic field) tend to the probability and to the slope of GT. Note that for circular polarization there is one group of four transitions and GT do not exist. Among eight transitions there are also two transitions (forbidden for $B = 0$) with the probabilities undergoing strong modification under the influence of magnetic fields. Practically the complete hyperfine Paschen-Back regime is observed at relatively low (~ 1 kG) magnetic fields. Note that for K D_2 line GT are absent. Theoretical models describe the experiment very well.

Introduction. – Atomic spectroscopy with NC filled with Rb or Cs atomic vapor, with a thickness of the vapor column L which is of the order of optical radiation wavelength has been found to be very efficient to study optical atomic transitions in external magnetic fields [1]. There are two interconnected effects: splitting of the atomic energy levels to Zeeman sublevels and shifting of frequencies (deviating from the linear dependence observed in quite moderate magnetic fields), and significant change in atomic transitions probabilities as a function of the B field [2–4]. These studies benefit from the following features of NC: i) sub-Doppler spectral resolution for atomic vapor thickness $L = \lambda/2$ and $L = \lambda$ (λ being the resonant wavelength of Rb $D_{1,2}$ or Cs $D_{1,2}$ lines) needed to resolve a huge number of Zeeman transition components in transmission or fluorescence spectra; ii) possibility to apply a strong magnetic field using permanent magnets in spite of a strong inhomogeneity of the B field (in our case it can reach 150 G/mm). Note that the variation of the B field inside the atomic vapor is negligible as the vapor column thickness is small. K vapor in magnetic fields was stud-

ied in [5] using a few centimeter-long cell and saturation absorption technique. However, due to the presence of strong cross-over resonances in the absorption spectrum, the technique is useful only for $B < 10$ G.

Here we report the first studies of K vapor confined in NC and under the influence of relatively low magnetic fields ($B < 2$ kG). The NC with $L = \lambda/2$ and λ are used for σ^+ and π laser polarizations. Also two theoretical considerations are taken into account. The splitting of atomic levels in weak magnetic fields is described by the total angular momentum $\mathbf{F} = \mathbf{J} + \mathbf{I}$ of the atom and its projection m_F , where $\mathbf{J} = \mathbf{L} + \mathbf{S}$ is the total angular momentum of electrons and \mathbf{I} is the nuclear spin. In the hyperfine Paschen-Back (HPB) regime, \mathbf{J} and \mathbf{I} become decoupled and the splitting of the atomic levels is described by the projections m_J and m_I . For alkali metals the hyperfine Paschen-Back regime takes place at fields $B \gg B_0 = A_{hfs}/\mu_B$, where A_{hfs} is the ground-state hyperfine coupling coefficient and μ_B is the Bohr magneton. For ^{133}Cs , ^{87}Rb , and ^{85}Rb atoms $B_0 \sim 1.7$ kG, 2 kG and 0.7 kG, respectively [6–10]. It is worth noting that ^{39}K

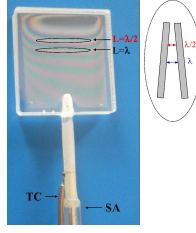


Fig. 1: The photograph of the K NC with a tapered gap of thickness L ranging from 50 to 1500 nm. TC is a thermocouple to measure the side-arm (SA) temperature.

has the smallest $B_0 \approx 0.17$ kG caused by the smallest hyperfine splitting $\Delta \approx 462$ MHz of the ground level $4S_{1/2}$ [note that $A_{hfs} = \Delta/(I + 1/2)$]. Thus, one expects to obtain complete J and I decoupling (HPB) of ^{39}K at relatively low magnetic fields $B \gg 170$ G. The manifestations of HPB regime, particularly, are as follows: i) strong reduction of the number of atomic transitions (compared with that at low magnetic field) to a fixed number which is easy to determine from the diagram on the basis of m_J and m_I projections, ii) the fixed frequency slope inside the group of transitions, iii) the transition probabilities tend to the same value inside the group, iv) the energy of the ground and upper levels can be calculated from analytic expression (1).

Experiment. — For our purpose a one-dimensional nano-metric-thin cell filled with natural potassium (93.3% ^{39}K , and 6.7% ^{41}K) has been built (for the first time) and used for the experiment. The design of the NC is similar to that of the extremely thin cell described earlier [11] (the details of NC design can be found in [12]). The NC allows one to exploit a variable vapor column thickness L in the range of 50 – 1500 nm. It is demonstrated that for K vapor the key parameter determining the spectral width and the shape of the absorption line in the NC is the ratio L/λ , with $\lambda = 770$ nm being the wavelength of the laser radiation resonant with the atomic transition of D_1 line (as it was earlier demonstrated for the NC filled with Cs or Rb). In particular, it was shown that the spectral width of the resonant absorption reaches its minimum value at $L = (2n + 1)\lambda/2$ (n is an integer); this effect has been called the Dicke-type coherent narrowing [12, 13, 15]. It is also demonstrated that for $L = n\lambda$ the spectral width of the resonant absorption reaches its maximum value, close to the Doppler width (> 0.9 GHz).

A photograph of the K NC with a tapered gap is shown in Fig. 1. The windows of the NC were constructed with well-polished crystalline sapphire with the c axis perpendicular to the window surface to minimize birefringence. The regions with the thickness $L = \lambda/2 = 385$ nm and $L = \lambda = 770$ nm are marked by ovals in Fig. 1. A sapphire side-arm (SA) filled with metallic K is seen at the bottom of Fig. 1. The SA was heated to 150 – 160 °C (the temperature on the windows was by 20 degrees higher in order

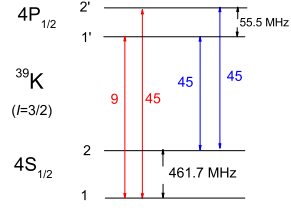


Fig. 2: The hyperfine (hfs) energy levels diagram of the D_1 line of ^{39}K (the prime is used for the upper levels).

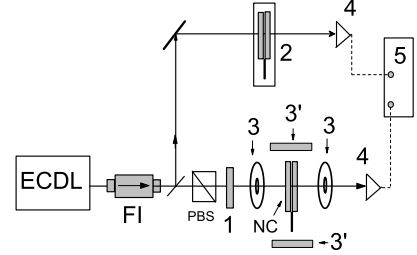


Fig. 3: Sketch of the experimental setup. ECDL, diode laser; FI, Faraday isolator; 1, $\lambda/4$ plate; main K NC with thickness $L = \lambda/2$ or $L = \lambda$ in the oven; PBS, polarizing beam splitter; 2, auxiliary K NC; 3, 3' permanent magnets: in case of σ^+ radiation use, the \mathbf{B} -field is directed along the laser propagation direction \mathbf{k} (magnets 3); for π polarization, the \mathbf{B} -field is directed along the laser electric field \mathbf{E} (magnets 3'); 4, photodetectors; 5, oscilloscope.

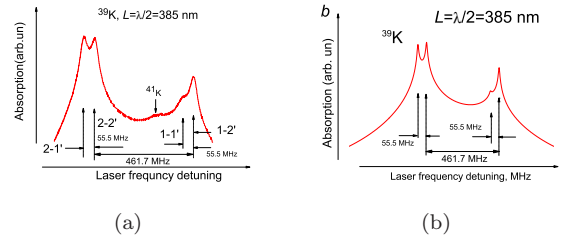


Fig. 4: Absorption spectrum of ^{39}K vapor contained in the NC $L = \lambda/2 = 385$ nm a) experiment, the laser power is $2 \mu\text{W}$, the SA temperature is 150 °C, a small absorption of ^{41}K is also seen; b) theory [16] with the parameters: thermal velocity $v_{th} = 450 \text{ ms}^{-1}$, Rabi frequency $\Omega/2\pi = 0.06\gamma_N$ ($\gamma_N \approx 6$ MHz).

to prevent vapor condensation) providing the density of atoms $N = (5 - 8) \times 10^{12} \text{ cm}^{-3}$. The NC can operate up to temperatures of 500 °C. The hfs energy levels diagram of the D_1 line of ^{39}K is shown in Fig. 2. A sketch of the experimental setup is shown in Fig. 3. The linearly polarized beam of an extended cavity diode laser ($\gamma_L < 1$ MHz), resonant with a ^{39}K D_1 line after passing through a Faraday isolator (FI), was focused onto a 0.5 mm diameter spot on the K NC orthogonally to the cell window. A PBS was

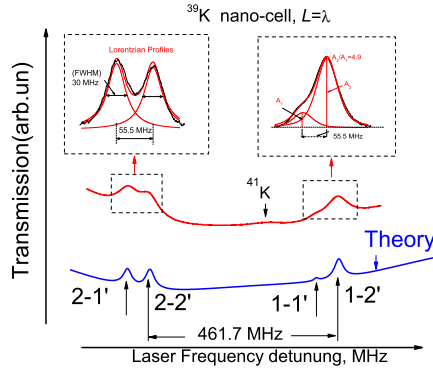


Fig. 5: Four VSOPs located at the positions of the transitions $1, 2 \rightarrow 1', 2'$ are presented in the transmission spectrum of ^{39}K vapor contained in the NC with $L = 770$ nm; a small absorption of ^{41}K is also seen; the SA temperature is 160°C . The inset shows the results of fitting by Lorentzian profiles; note that the ratio of the VSOPs amplitudes coincides with that deduced from the values given in Fig. 2 which means linear dependence on laser power. The lower curve - the theory [16] for the parameters: $v_{\text{th}} = 450 \text{ ms}^{-1}$, Rabi frequency $\Omega/2\pi = 0.2\gamma_N$.

used to purify the initial linear polarization of the laser; a $\lambda/4$ plate (1) was used to produce a circular polarization. In the experiments the thicknesses of the vapor column $L = \lambda$ and $L = \lambda/2$ were used. The transmission signal was detected by a photodiode (4) and was recorded by a Tektronix TDS 2014B four-channel storage oscilloscope (5). To record the transmission spectra, the laser radiation was linearly scanned within up to a ~ 5 GHz spectral region covering the studied group of transitions. About 30% of the laser power was branched to the reference unit with an auxiliary K NC (2). The absorption spectrum of the latter with thicknesses $L = \lambda/2$ or $L = \lambda$ was used as frequency reference.

The experimental and theoretical absorption spectra of K atomic vapor contained in the NC with $L = \lambda/2 = 385$ nm are shown in Fig. 4 (the large width of the spectrum at the base is caused by huge Doppler-broadening > 0.9 GHz at 170°C at the NC windows). There is a strong spectral narrowing which allows one to separate four transitions between hyperfine sublevels shown in Fig. 2. The sharp (nearly Gaussian) absorption near the top makes it convenient to separate closely spaced individual atomic transitions in an external magnetic field (we called it $\lambda/2$ -method [10]). We have also used the so called λ -method [15, 16]. In this case spectrally narrow velocity selective optical pumping (VSOP) resonances located exactly at the positions of atomic transitions appear in the transmission spectrum of the NC with thickness $L = \lambda$ shown in Fig. 5. The VSOP parameters are shown to be immune against 10% thickness deviation from $L = \lambda$, which makes the λ -method feasible. Laser power is $\sim 15 \mu\text{W}$ (for a lower power the VSOP line-width is less

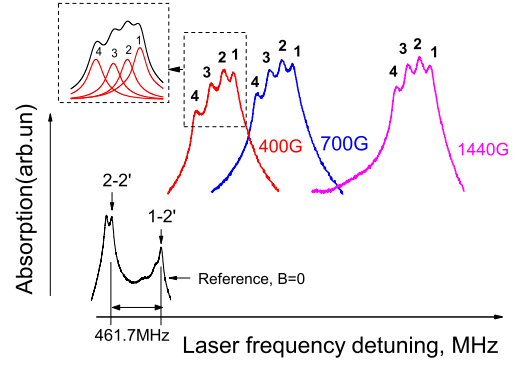


Fig. 6: Absorption spectrum of ^{39}K vapor contained in the NC with $L = \lambda/2$ for $B = 400, 700$ and 1440 G; σ^+ excitation. The bottom curve is the reference showing the positions of ^{39}K transitions for $B = 0$. The inset shows the results of fitting by four "pseudo-Vigt" functions (note that the amplitude of the transition labeled 4 is larger than that of 3). The absolute value of the peak absorption of transition 1 is $\sim 0.3\%$.

than 30 MHz, but is more noisy). In a magnetic field, the VSOPs are split into many components. The amplitude and frequency positions of VSOPs depend on the B -field, which makes it convenient to study each individual transition separately [1].

Magnetometry with $\lambda/2$ - and λ -methods, σ^+ laser excitation. — The assembly of an oven with NC inside was placed between the permanent ring magnets. The magnetic field \mathbf{B} was directed along the laser radiation propagation direction \mathbf{k} . An extremely small thickness of the NC is advantageous for the application of strong magnetic fields with the use of permanent magnets (having a 2 mm diameter hole for the laser beam passage). Such magnets are unusable for centimeter-long cells because of the strong inhomogeneity of the magnetic field, while in NC, the variation of the B -field inside the atomic vapor column is by several orders less than the applied B value. The B -field strength was measured by a calibrated Hall gauge with an absolute imprecision less than 30 G throughout the applied B -field range. The absorption and transmission spectra of ^{39}K vapor contained in the NC with $L = \lambda/2$ and $L = \lambda$ (for σ^+ radiation) versus magnetic field are shown in Fig. 6 and Fig. 7, respectively (the bottom curves are the reference ones). Four atomic transitions (as predicted for the HPB regime by the diagram presented in Fig. 8) are well observable. Note that the frequency separation between the two well resolved VSOP labeled 1 and 2 is about 90 MHz, which is more than 10 times less than the Doppler line-width. Also, the VSOPs have the same amplitudes which is an additional evidence of HPB regime. Let us briefly stress the benefits of $\lambda/2$ - and λ -methods. The $\lambda/2$ -method requires less laser intensity and in case of small absorption the peak absorption A is proportional to σNL , where σ is the ab-

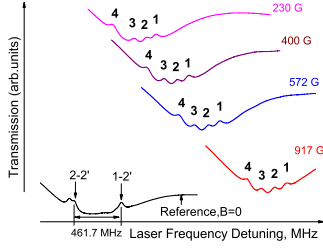


Fig. 7: Transmission spectrum of ^{39}K vapor contained in the NC with $L = \lambda$ for $B = 230, 400, 572$ and 917 G and σ^+ excitation. The bottom curve is the reference showing the positions of ^{39}K transitions for $B = 0$.

sorption cross section proportional to d^2 (with d being the dipole moment). Thus, by direct comparison of A_i (peak amplitudes of the absorption of the i -th transition), it is straightforward to estimate the relative probabilities (i.e. line intensities). On the other hand, the λ -method provides a fivefold better spectral resolution. Thus, the methods can be considered as complementary depending on particular requirements. Moreover, it is easy to switch from $\lambda/2$ to λ just by vertical translation of the NC. Note, that in the case where there is a big frequency separation between the transitions (as it is for the ^{87}Rb isotope) also a 1 mm long cell can be used [9, 17].

Theoretical simulations for the B -field dependences of the atomic transition frequency shifts and relative transition probabilities for $1, 2 \rightarrow 1', 2'$ transitions of the K D_1 line were based on the calculation of the eigenvalues and eigenvectors of the Hamilton matrix versus magnetic field for the full hyperfine structure manifold [2, 4, 18]. Although for the K D_1 line the Rabi-Breit formula could be also used, however the theoretical model is preferable since it is valid also for K D_2 line. Note, that in the case of HPB regime the energy of the ground $4S_{1/2}$ and upper $4P_{1/2}$ levels for the ^{39}K D_1 line is given by the following formula [2, 4]:

$$E_{|Jm_JIm_I\rangle} = A_{hfs}m_Jm_I + \mu_B(g_Jm_J + g_I m_I)B. \quad (1)$$

The values for the fine structure (g_J) and the nuclear (g_I) Landé factors and the hyperfine constants A_{hfs} are given in [21]. The dependence of the frequency shifts on the magnetic field (relative to the position of the $2 \rightarrow 2'$ transition at $B = 0$) is shown in Fig. 9, where the red lines are obtained by exact numerical calculation, while the black lines are plotted with formula (1); the black squares are the experimental results. The inaccuracy does not exceed 2%. A good agreement of theory and experiment is observed. The inset shows the residual difference between numerics (red lines) and calculations (black lines) with formula 1. It is remarkable that at a relatively low magnetic field ~ 1000 G formula (1) describes the transition frequency values with an inaccuracy of $\leq 1\%$ (while at $B = 1.8$ kG the difference is $\leq 0.3\%$ which could be considered as practically the full HPB regime). Fig. 10 presents

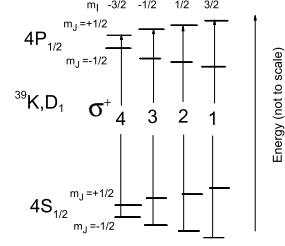


Fig. 8: Diagram of ^{39}K D_1 line transitions in HPB regime for σ^+ laser excitation. The selection rules: $\Delta m_J = 1$; $\Delta m_I = 0$.

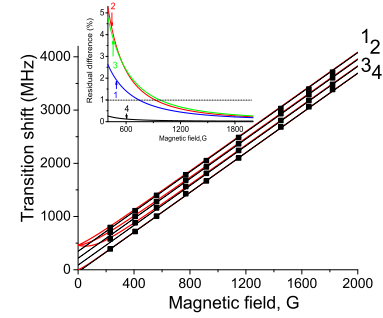


Fig. 9: B -field dependence of the frequency shifts for transitions 1- 4 of the ^{39}K D_1 line in case of σ^+ excitation. Red lines are given by exact numerical calculations, black solid lines by formula (1), black squares by experiment. The inaccuracy is $\sim 2\%$. Inset presents the residual difference between numerics and calculations by formula (1) for curves 1-4.

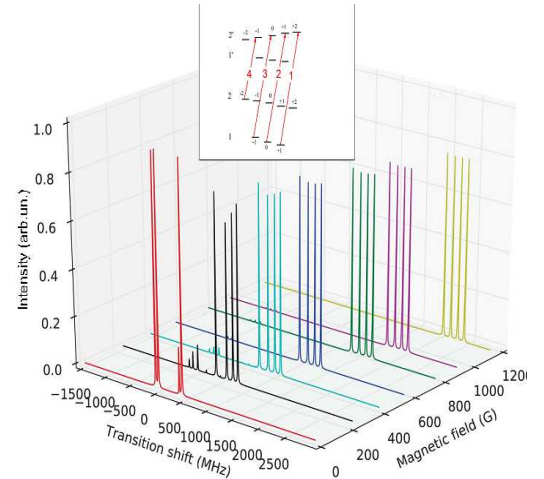


Fig. 10: The intensities and frequencies shifts (theory) of the ^{39}K transitions versus B -field for σ^+ excitation. As seen for $B > B_0$ only 1-4 transitions remain and the probabilities tend asymptotically to the same value at $B \gg B_0$. The inset shows F, m_F for the ground and the upper levels for transitions 1-4 .

the theoretical values of the atomic transition probabilities (intensities) as a function of the B -field. As seen the intensities of transitions 1-4 at $B \geq 1$ kG tend to the same asymptotic value (HPB regime), which coincides with the experiment, while at $B < 400$ G, the intensity of the transition labeled 4 (Fig. 10) is larger than those of 3 and 2 which also coincides with the experiment.

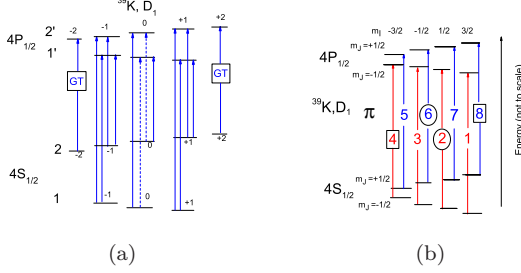


Fig. 11: a) Diagram of the hfs energy levels of D_1 line of ^{39}K in a magnetic field (for $B < B_0$) and possible Zeeman transitions for π -polarized excitation are shown. The selection rules are $\Delta F = 0, \pm 1$; $\Delta m_F = 0$. GT are labeled by rectangles; two IFFA transitions are indicated by dashed lines; b) Diagram of the $K D_1$ line transitions in the HPB regime for π -polarised excitation. The selection rules are: $\Delta m_J = 0$; $\Delta m_I = 0$. Two IFFA transitions are labeled by ovals.

Magnetometry with $\lambda/2$ - and λ -methods, π -polarised excitation. — For this case the \mathbf{B} -field is directed along the laser electric field \mathbf{E} (magnets 3' are used and $\lambda/4$ plate is removed). Fig. 11(a) shows the hfs energy levels diagram of the D_1 line of ^{39}K atoms in a magnetic field ($B < B_0$) and 14 possible atomic Zeeman transitions for π -polarized excitation [including two forbidden at $B = 0$ atomic transitions [21, 22] shown by dashed lines that we call "initially forbidden further allowed" (IFFA)]. Fig. 11(b) presents the diagram of eight remaining transitions (the probabilities increase with the B -field).

Note, that the necessary conditions for a modification

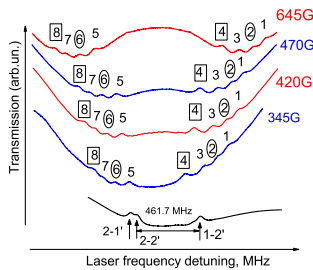


Fig. 12: Transmission spectrum of the NC with ^{39}K for $L = \lambda$ for $B = 345, 420, 470$ and 645 G; π -polarized excitation is used. GT and IFFA transitions are labeled by rectangles and ovals, respectively. The bottom curve is the reference.

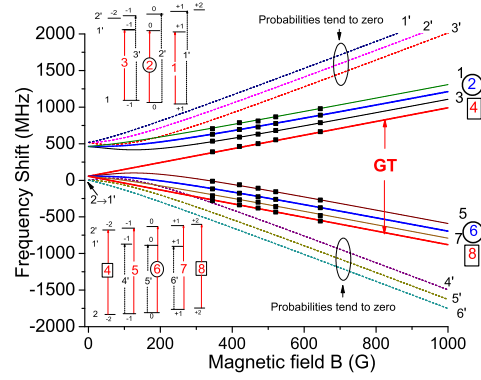


Fig. 13: B -field dependence of the frequency shifts for transitions 1-8 of ^{39}K D_1 line for π -polarized excitation. GT and IFFA transitions are marked by rectangles and ovals, respectively. The probabilities of six transitions (dashed lines) tend to zero for $B > B_0$. Solid lines - numerics; symbols - experiment (the inaccuracy is $\sim 2\%$). The insets show the initial quantum numbers F, m_F for the ground and upper levels.

of the probability are the following [18, 19]: the perturbation induced by the B -field couples only sublevels with $m_F - m_F' = 0$ which satisfy the selection rules $\Delta L = 0$, $\Delta J = 0$, $\Delta F = 1$, and modification of the probability is possible only for transitions between ground and excited sublevels when at least one of them is coupled with another transition's sublevel according to the selection rules. As seen from Fig. 11(a), for the two side atomic transitions of K (marked as GT), $|2, m_F = +2\rangle \rightarrow |2', m_F' = +2\rangle$ and $|2, m_F = -2\rangle \rightarrow |2', m_F' = -2\rangle$ the sublevels which could be mixed according to the selection rules are absent. Thus the probability of these two transitions remain the same in the whole range of applied B -fields, while the probabilities of other transitions differ significantly at low fields, but tend to the same value within the group at $B \gg B_0$. It is remarkable that the probabilities of all the other atomic transitions tend to that of two side atomic transitions (this is confirmed by the numerical calculations). For this reason why we call them guiding transitions. In Fig. 12 the transmission spectra of π -polarized excitation of the potassium NC ($L = \lambda$ for $B = 345, 420, 470$ and 645 G) are shown. GT and IFFA transitions are labeled by rectangles and ovals, respectively (as seen IFFA transitions undergo strong modification under the influence of the B -field).

The B -field dependence of the frequency shifts for transitions 1-8 for π -polarized radiation is shown in Fig. 13. GT and IFFA transitions are labeled by rectangles and ovals, respectively. We see that transitions 1-8 are contained in two groups of four atomic transitions each. Note that the frequency slope (s) of four transitions inside the group asymptotically tends to the slope of GT. It is easy to show that they are equal to $s = 0.94$ MHz/G and $s = -0.94$ MHz/G for groups 1-4 and 5-8, respectively.

The intensity and the frequency shifts (theory) of the K

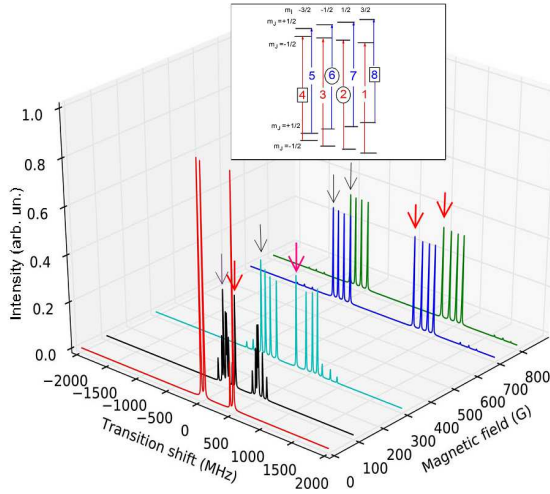


Fig. 14: Theoretical curves for intensities and frequency shifts of the atomic transitions of ^{39}K D_1 line vs B -field for π -polarized excitation. As seen for $B \gg B_0$ only 1-8 transitions remain and the probabilities tend asymptotically to the same value as for GT transitions (by module 4.66×10^{-18} ESU). Groups 1-4 and 5-8 are located at the high and low frequency side, respectively, while the transitions labeled 1 and 8 have the largest and smallest frequencies, respectively. GT are marked by arrows (each by the same colour) and as seen their amplitudes remain the same in the range of 100 – 800 G.

transitions vs B -field for π -polarized excitation are shown in Fig. 14. As seen for $B \gg B_0$ only transitions 1-8 remain and the probabilities at $B \gg B_0$ tend asymptotically to the same value as GT (by module 4.66×10^{-18} ESU). The inset shows the diagram of the K D_1 line transitions in HPB regime with GT and IFFA marked.

Summary and conclusions. – It is remarkable that with K atoms (due to the small value of B_0 for ^{39}K) contained in NC all the peculiarities of the behavior of atomic transitions of the Cs, ^{87}Rb , and ^{85}Rb in the whole region of B -field up to the HPB regime could be easily studied using smaller, at least by an order, B -fields. For example, in Ref. [23] the ^{87}Rb D_1 line for π -polarized excitation was studied (the diagram is similar to the one shown in Fig. 11(a)). For a B -field of 220 G there was no any evidence of IFFA transitions, while for the K atoms IFFA transitions (shown in Fig. 12) have amplitudes comparable with the other transitions. Also, the peculiarities of the HPB regime for the K atoms can be detected at B -fields smaller by at least one order. Note that in [1] a magnetometer based on the Rb NC use was described, while the spatial resolution of K NC for $L = \lambda/2 = 385$ nm should be better than that of the Rb (398 nm) and Cs (448 nm), which is important when measuring a strongly inhomogeneous magnetic field. It is worth to note that in some cases instead of Rb NC a micrometric-thin ($\sim 40 \mu\text{m}$) cell could be successfully used [24]. Thus, we suppose that

with the help of a $40 \mu\text{m}$ -thin K cell the above mentioned peculiarities could be studied, too.

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